Year 9 Headstart Mathematics

Applied Trigonometry

Term 4 – Week 4

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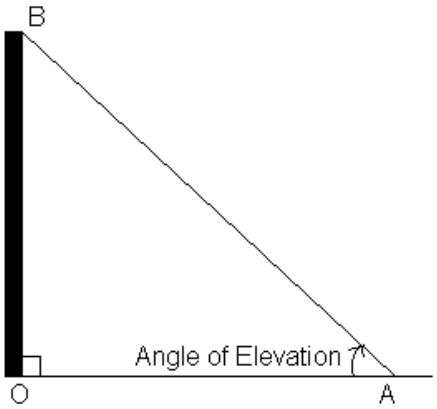
Term 4 – Week 4 – Theory

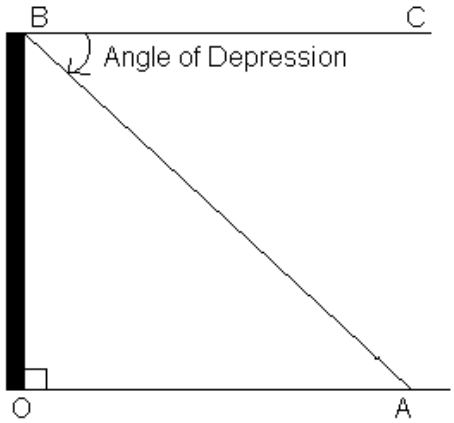
Angles of Elevation and Depression:

Angle of Elevation:



The angle of elevation from A to B is the angle between the line OA and AB. $\angle OAB$ is the angle of elevation.

Angle of Depression:



The angle of depression from B to A is the angle between the line BC and BA. $\angle CBA$ is the angle of depression.
(NOTE: the angle of depression $\angle CBA$ is the same as the angle of elevation $\angle OAB$ because OA||BC, hence they are alternate angles).

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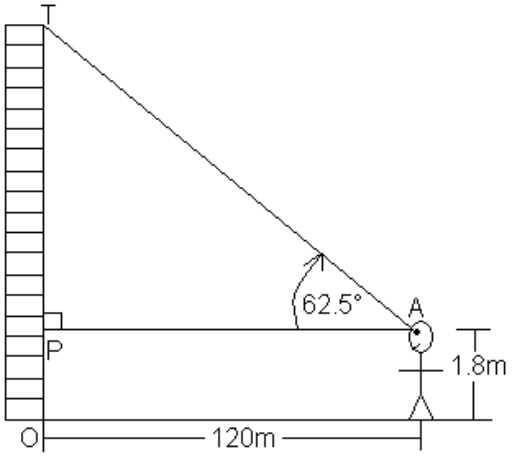
Example:

A man 1.8 metre tall is 120 metres from the base of a building. The man looked up to the top of the building and measured the angle of elevation to be $62.5°$, find the height of the building and correct to the nearest 2 decimal places.

Solution:



$$\tan 62.5° = \frac{PT}{PA}$$

$$\tan 62.5° = \frac{PT}{120}$$

$$PT = 120 \tan 62.5°$$
$$PT = 230.52$$

$\therefore$ The height of the building
$= 230.52 + 1.8$
$= 232.32$ metres

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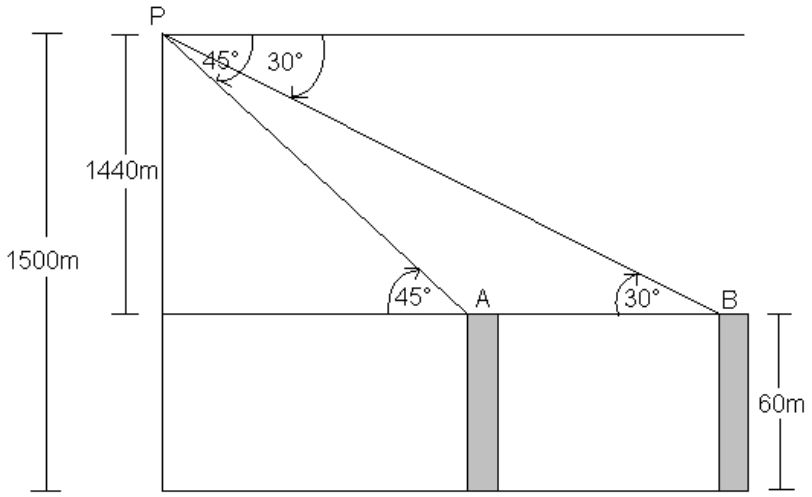
Example:

The angle of depression from an airplane to the tops of two building A and B (in line with the airplane) are $45°$ and $30°$ respectively. The airplane is 1500m above ground and the two buildings each has height of 60m. Find the distance between the two buildings.

Solution:



$$\tan 30° = \frac{1440}{OB}$$

$$OB = \frac{1440}{\tan 30°}$$

$$= \frac{1440}{\frac{1}{\sqrt{3}}}$$

$$OB = 1440\sqrt{3}\text{ m}$$

$$\tan 45° = \frac{1440}{OA}$$

$$OA = \frac{1440}{\tan 45°}$$

$$= \frac{1440}{1}$$

$$OA = 1440\text{ m}$$

$$AB = OB - OA$$
$$= 1440\sqrt{3} - 1440$$
$$\therefore AB = 1054.15\text{m (to the nearest 2 d. p)}$$

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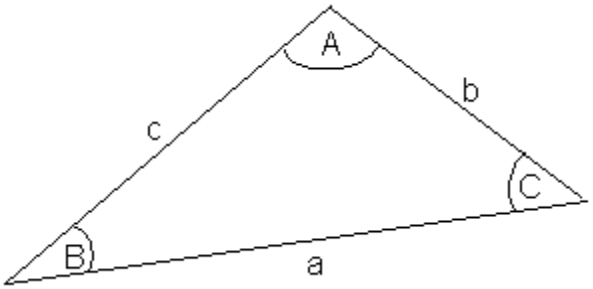
Sine Rule, Cosine Rule and Area of a Triangle:

The Sine Rule:



The Sine Rule states:

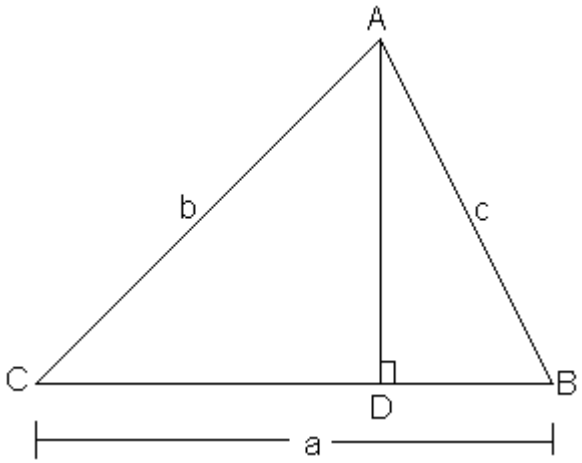
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Or,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

(NOTE: Sometimes, the **ambiguous case** arises when using the sine rule. This occurs when there are two possible angles for the unknown angle which means two possible triangles can be constructed. The unknown angle can be both acute and obtuse).

Proof:



Construct AD perpendicular to BC.

In ΔABD, $\frac{AD}{c} = \sin B$

So, $AD = c.\sin B$

In ΔACD, $\frac{AD}{b} = \sin C$

So, $b = \frac{AD}{\sin C}$

$$b = \frac{c.\sin B}{\sin C}$$

$$\therefore \frac{b}{\sin B} = \frac{c}{\sin C}$$

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Similarly, $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

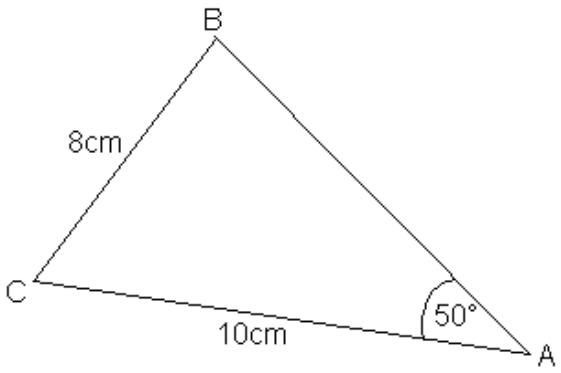
OR equivalently,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example:

In ΔABC, $\angle A = 50°$, $BC = 8cm$ and $AC = 10cm$, find the magnitude of $\angle B$.

Solution:



$$\frac{\sin B}{10} = \frac{\sin 50°}{8}$$

$$\sin B = \frac{10 \sin 50°}{8}$$

$$B = \sin^{-1}\left(\frac{10 \sin 50°}{8}\right)$$

$$\therefore B = 73°15' \text{ or } 106°45'$$

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Term 4 – Week 4 – Homework

Angles of Elevation and Depression:

1. A person 180cm tall standing on the ground is observing the top of a flagpole. If the person is 8m from the base of the flagpole and the angle of elevation of the top of the flagpole is $60°$, find the height of the flagpole to the nearest 0.1m.

2. A person in a boat is 115m away from the base of a lighthouse. If the angle of elevation of the top of the lighthouse is $20°$, find the height of the lighthouse to the nearest 3 significant figures.

3. A person in a hot air balloon moving in a horizontal straight line at a height of 500m passes directly over Central Station. Ten minutes later, the person finds the angle of depression of Central Station is $35°26'$. Find:
   (i) The distance traveled by the hot air balloon in that time.
   (ii) The speed of the hot air balloon in m/min and km/h.

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4. From a helicopter flying at a height of 1500m above the ground, the angles of depression of point A and B on the ground are $35°$ and $70°$ respectively. Find the distance between the two points to the nearest 10m.

5. From the top of tower A, the angle of depression of the top of tower B is $26°20'$. If the heights of tower A and B are 120m and 85m respectively, find the distance between the two towers correct to the nearest 2 decimal places.

6. P and R are the bases of two vertical buildings PQ and RS respectively. The angle of elevation from R to Q is $15°$ and the angle of elevation from P to S is $35°$. If the height of the building PQ is 80m, find:
   (i) The distance between P and R.
   (ii) The height of the building SR.
   (iii) The angle of depression from S to Q.

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7. A person standing on top of a building with height 50m is observing a sign on the wall of another building. The angle of depression from the person to the sign is $45°$. If the horizontal distance between the two buildings is 30m, find:
   (i) The height of the sign above the ground.
   (ii) The angle of depression from the sign to the base of the first building (correct to the nearest minute).

8. Two vertical towers of unequal height are 60m apart. The angles of elevation and depression from the top of the shorter tower to the top and bottom of the taller tower are $38°$ and $52°$ respectively. Find:
   (i) The height of the shorter tower.
   (ii) The height of the taller tower.
   (iii) The angle of depression from the top of the taller tower to the base of the shorter tower.

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9. From the top of a vertical tower with height 65m, the angle of depression to a point A due east of it is $35°$ and the angle of depression to another point B due west of it is $75°$. Find the distance between the two points.

10. The angles of depression from a 50m tall building to the points P and Q on the ground is $60°$ and $45°$. Find the distance between the points P and Q in exact values.

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The Sine Rule:

1. In ΔABC, $\angle A = 60°$, $\angle B = 30°$ and AB is 10cm, find the magnitude of $\angle C$ and the length of AC.

2. In ΔABC, $\sin B = \frac{3}{5}$, $BC = 8m$ and $AC = 12m$, find the magnitude of $\angle A$ and the length of AB.

3. In ΔABC, $\angle B = 20°$, $AC = 12cm$ and $BC = 16cm$, find the magnitude of $\angle A$.

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4. In ΔPQR, $\angle R = 10°$, $QR = 21m$ and $PQ = 8m$, find the magnitude of $\angle P$ and the length of PR.

5. Two boats, A and B, leave a port O. Boat B travels 30m in a direction $125°\text{T}$ and boat A travels in a direction of $175°\text{T}$. If the final distance between boats A and B is 25m, find the bearing of boat A from boat B.

6. A cyclist travels 5km from home to the grocery shop in a direction $30°\text{T}$. He then travels 4km to the post office which is due north of his home. Find the distance between the post office and his home.

End of homework

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