Year 10 Mathematics

Curve Sketching and Using Curves

Term 2 – Week 3

Student Name: ……………………………….

Class code: …………………………………..

Teacher name: ……………………………..

DUX College

T: (02) 8007 6824
E: info@duxcollege.com.au
W: www.duxcollege.com.au

Term 2 – Week 3 – Theory

Domain and Range:

The **domain** of a function is the set of all first elements of the ordered pairs. In other words, domain is the set of all possible x-values.

The **range** of a function is the set of all second elements of the ordered pairs. In other words, range is the set of all possible y-values.

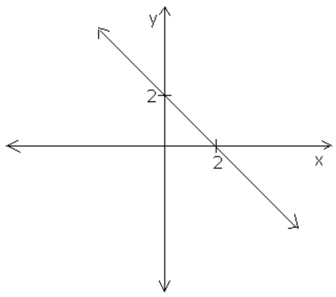
Example:

Sketch the following functions then determine the domain and range.

(i) $x + y - 2 = 0$
(ii) $y = x^2 + 2$
(iii) $y = x^2 - 6x + 5$
(iv) $x^2 + y^2 = 16$
(v) $y = \sqrt{4 - x^2}$

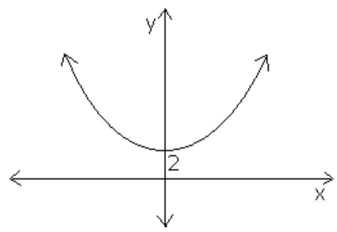
Solution:

(i) $x + y - 2 = 0$



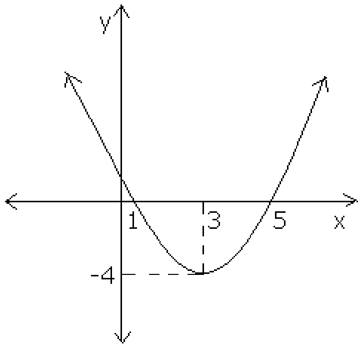
Domain: All real x
Range: All real y

(ii) $y = x^2 + 2$



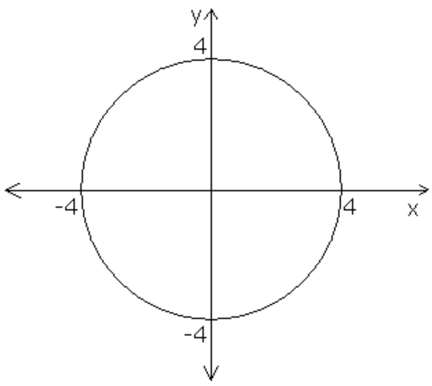
Domain: All real x
Range: $y \geq 2$

(iii) $y = x^2 - 6x + 5$



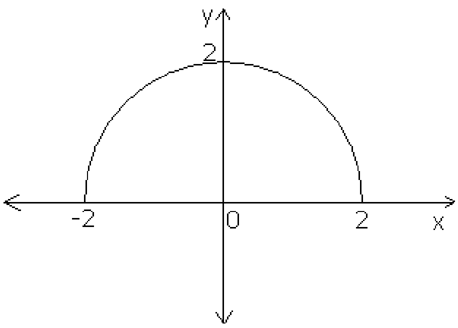
Domain: All real x
Range: $y \geq -4$

(iv) $x^2 + y^2 = 16$



Domain: $-4 \leq x \leq 4$
Range: $-4 \leq y \leq 4$

(v) $y = \sqrt{4 - x^2}$



Domain:$-2 \leq x \leq 2$
Range: $0 \leq y \leq 2$

Domain and Range for Hyperbolic Functions:

Hyperbolic functions are in the form of $y = \frac{k}{x}$. It is obvious that the denominator of the function can never equal to zero because, if it does, then the function is undefined. Hence, the domain is restricted by the set of points which makes the denominator equal to zero.

Example:

Find the domain and range for the following hyperbolic functions.

(i) $y = \frac{1}{x-3}$
(ii) $y = -\frac{1}{2x+5}$
(iii) $y = \frac{4}{3-5x} + 1$
(iv) $y = \frac{1}{x^2+9x+14}$
(v) $y = \frac{1}{4-3x-x^2}$

Solution:

(i) $y = \frac{1}{x-3}$

Domain: $x - 3 \neq 0$

$x \neq 3$

$\therefore$ All real x, but $x \neq 3$

Range: $y \neq 0$

(ii) $y = -\frac{1}{2x+5}$

Domain: $2x + 5 \neq 0$

$x \neq -\frac{5}{2}$

$\therefore$ All real x, but $x \neq -\frac{5}{2}$

Range: $y \neq 0$

(iii) $y = \frac{4}{3-5x} + 1$

Domain: $3 - 5x \neq 0$

$x \neq \frac{3}{5}$

$\therefore$ All real x, but $x \neq \frac{3}{5}$

Range: $y \neq 1$

(iv) $y = \frac{1}{x^2+9x+14}$

Domain: $x^2 + 9x + 14 \neq 0$

$(x+2)(x+7) \neq 0$

$x \neq -2, -7$

$\therefore$ All real x, but $x \neq -2\,, -7$

Range: $y \neq 0$

(v) $y = \frac{1}{4-3x-x^2}$

Domain: $4 - 3x - x^2 \neq 0$

$(4+x)(3-x) \neq 0$

$x \neq 3, -4$

$\therefore$ All real x, but $x \neq 3\,, -4$

Range: $y \neq 0$

Domain and Range for Square Root Functions:

Square root functions are in the form of $y = \sqrt{x}$. It is obvious that the value of x must be greater than or equal to zero. Otherwise, square root of a negative number has no real values. Hence, the domain of x must be greater than or equal to zero.

Example:

Find the domain and range for the following functions.

(i) $y = \sqrt{x}$
(ii) $y = -\sqrt{1-2x}$
(iii) $y = \sqrt{x^2 - 9x - 10}$
(iv) $y = \sqrt{7 - 6x - x^2}$

Solution:

(i) $y = \sqrt{x}$

Domain: $x \geq 0$

Range: $y \geq 0$

(ii) $y = -\sqrt{1-2x}$

Domain: $1 - 2x \geq 0$

$-2x \geq -1$

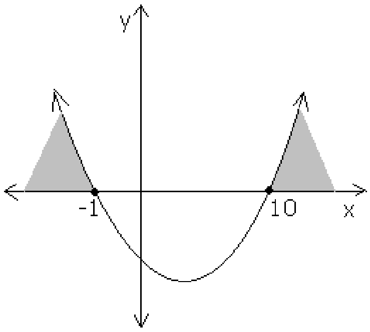
$\therefore x \leq \frac{1}{2}$

Range: $y \leq 0$

(iii) $y = \sqrt{x^2 - 9x - 10}$

Domain: $x^2 - 9x - 10 \geq 0$

$(x - 10)(x + 1) \geq 0$



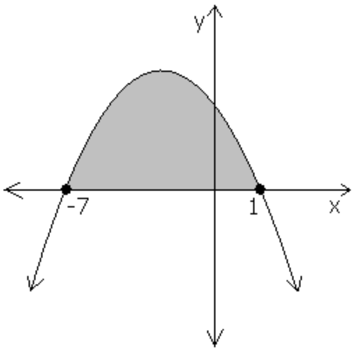
$\therefore x \leq -1, x \geq 10$

Range: $y \geq 0$

(iv) $y = \sqrt{7 - 6x - x^2}$

Domain: $7 - 6x - x^2 \geq 0$

$(7 + x)(1 - x) \geq 0$



$\therefore -7 \leq x \leq 1$

Range: $y \geq 0$

Term 2 – Week 3 – Homework

Domain and Range:

1. Find the domain and range for the following functions. (It would be helpful by sketching them first).

a) $y = x + 2$
b) $y = 4 - 2x$
c) $2x - y = 4$
d) $3y + 2x - 12 = 0$
e) $y = x^2$
f) $x^2 + y^2 = 4$
g) $y = -\sqrt{1 - x^2}$
h) $y = -x^2$
i) $y = x^2 + 1$
j) $x^2 + y^2 = \frac{4}{9}$
k) $y = (x + 3)^2$
l) $y = x^2 - 4x + 3$
m) $y = \sqrt{16 - x^2}$
n) $x = \sqrt{8 - y^2}$

2. Find the domain and range for (a) to (h) and find the domain only for (i) to (p).

a) $y = \frac{1}{x}$
b) $y = \frac{1}{x-1}$
c) $y = -\frac{1}{x+2}$
d) $y = \frac{2}{5-x}$
e) $y = -\frac{1}{2x-1}$
f) $y = \frac{1}{3x+1}$
g) $y = \frac{3}{4-3x}$
h) $y = -\frac{1}{2-5x} + 1$
i) $y = \frac{1}{x^2-4} + 2$
j) $y = \frac{10}{16-x^2} - 3$
k) $y = -\frac{1}{x^2+2x+1}$
l) $y = \frac{3}{x^2-7x+12}$
m) $y = \frac{-2}{x^2+2x-15}$
n) $y = \frac{1}{2x^2+9x-5}$
o) $y = \frac{1}{3x^2+x-2}$
p) $y = \frac{1}{6+x-2x^2}$

3. Find the domain and range for the following functions.

a) $y = \sqrt{x}$
b) $y = -\sqrt{x}$
c) $y = \sqrt{x - 5}$
d) $y = -\sqrt{x + 2}$
e) $y = \sqrt{1 - x}$
f) $y = -\sqrt{2x - 3}$
g) $y = \sqrt{5x + 2}$
h) $y = \sqrt{2 - 3x}$
i) $y = -\sqrt{4 - 5x}$
j) $y = \sqrt{x^2 + 3x + 4}$
k) $y = \sqrt{x^2 - 5x - 6}$
l) $y = -\sqrt{x^2 - 2x - 35}$
m) $y = \sqrt{6 - x - x^2}$
n) $y = -\sqrt{15 + 2x - x^2}$
o) $y = \sqrt{4x^2 + 4x - 3}$
p) $y = \sqrt{2 + x - 6x^2}$

End of homework