Headstart Science

Energy, Forces and Motion II

Term 1 – Week 4

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TERM 1 – WEEK 4 – THEORY

GRAPHING MOTION

ANALYSING MOTION USING GRAPHS

In many situations, motion is represented by graphs. It is necessary for students to understand the features of motion graphs, as well as perform calculations on motion graphs. Below are two common types of graphs, the distance-time graph and the speed-time graph.

DISTANCE-TIME GRAPH

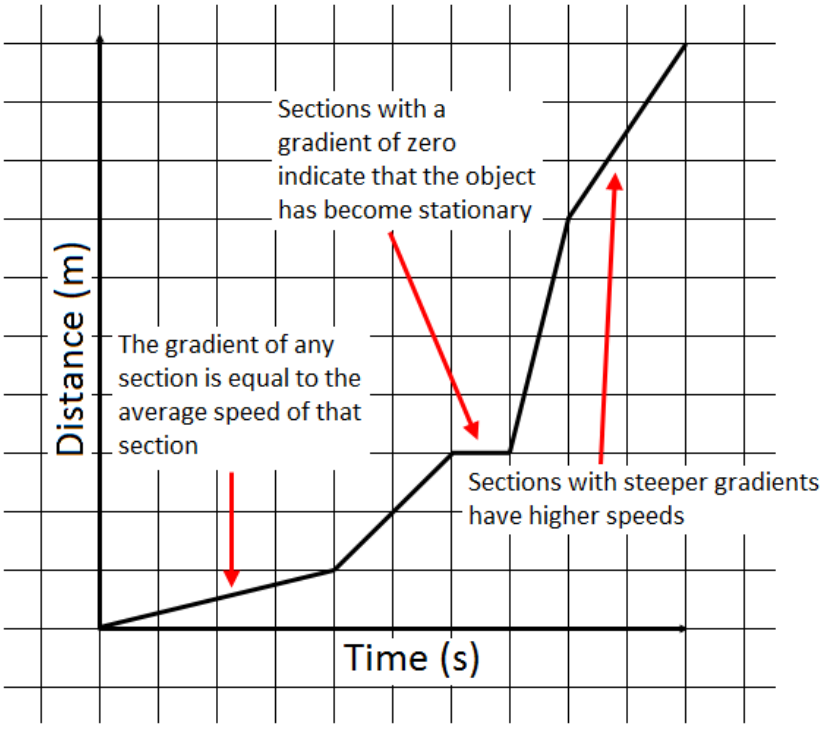
A distance-time graph shows the **total distance travelled by an object at various times in its motion**. Time is usually measured in seconds and placed on the horizontal axis. Distance is usually measured in metres and placed on the vertical axis.

The gradient of the graph is equal to:

$$Gradient = \frac{distance}{time} = speed$$

The **greater the gradient** of a distance-time graph**, the higher the instantaneous speed** of that object at a particular point in time. A horizontal line in the distance-time graph has a gradient of zero. Therefore, a **horizontal line** in this type of graph means that the **object is stationary**.

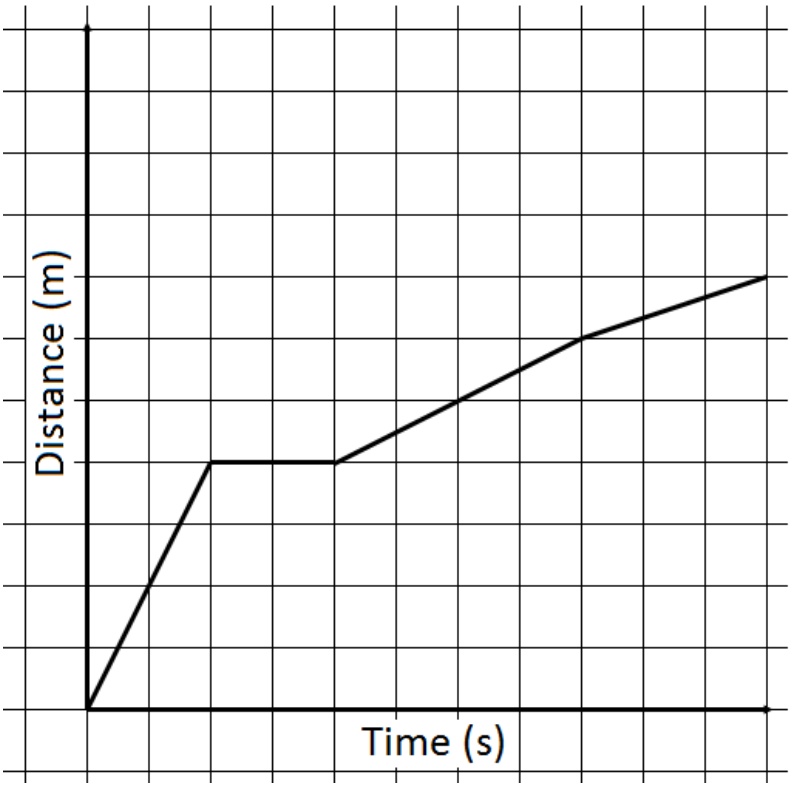
The graph below summarises some of the features of distance-time graphs.



Below is an example of various questions which can be asked from a simple distance-time graph.

Example 1

A man walked along a straight path. His motion is represented by the graph below.



Each side of a square represents one unit (second or metre).

Using the above graph, answer the following questions:

a) Identify the distance the man has travelled after two seconds of walking.
b) At what period of time is the man stationary?
c) Calculate the average speed the man is walking at between the fourth and eighth seconds of motion.
d) During which period of time is the man moving at the fastest speed, and what is the speed at this time?
e) Calculate the average speed of the man's entire journey.
f) Describe what is happening throughout this journey.

Solution

a) Find the point on the graph which represents a time of two seconds. At this point, the distance travelled by the man is 4 metres.

b) The gradient of a distance-time graph represents the speed of the man. The man will be stationary when the gradient of the graph is zero.

Therefore, the man is stationary between the second and fourth seconds of motion.

c) Recall from above that:

$$Speed = Gradient$$

In this example, for the motion between the fourth and eighth seconds:

$$Gradient = \frac{6-4}{8-4} = 0.5\ m/s$$

Therefore, the average speed of the man in this interval of time is 0.5 m/s.

d) Since speed is equal to the gradient of the distance-time graph, it follows that the section of the graph with the largest gradient will be the time where the man is travelling at the highest speed.

The first section of the graph, between zero second and two seconds, has the steepest gradient, and will therefore be the period of time where the man is moving at the fastest speed.

$$Speed = Gradient = \frac{4-0}{2-0} = 2\ m/s$$

e) Calculating average speed over the entire journey is calculated the same way as calculating the average speed over a section of the graph.

$$Average\ Speed = Gradient = \frac{7-0}{11-0} = 0.64\ m/s$$

f) The man initially walks at a speed of 2 m/s for two seconds, and then stops for the next two seconds. After this, the man continues walking at a lower speed for four seconds and then walks for a further three seconds at a slower pace.

SPEED-TIME GRAPH

A speed-time graph shows the instantaneous speed of an object at various times in its motion. Time is usually measured in seconds and placed on the horizontal axis. Speed is usually measured in metres per second and is placed on the vertical axis.

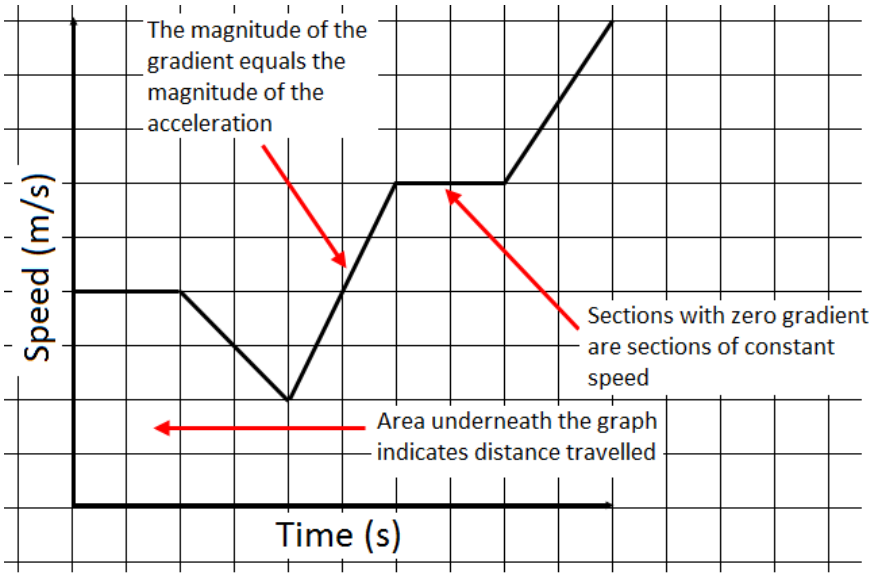
The gradient of the graph is equal to:

$$Gradient = \frac{change\ in\ speed}{time\ taken} = acceleration$$

Therefore, **the steeper the gradient** of any section of the speed-time graph, **the greater the acceleration** that an object experiences for that section of the graph. Sections of the graph that are flat and therefore have a gradient of zero are sections with constant speed – acceleration is zero.

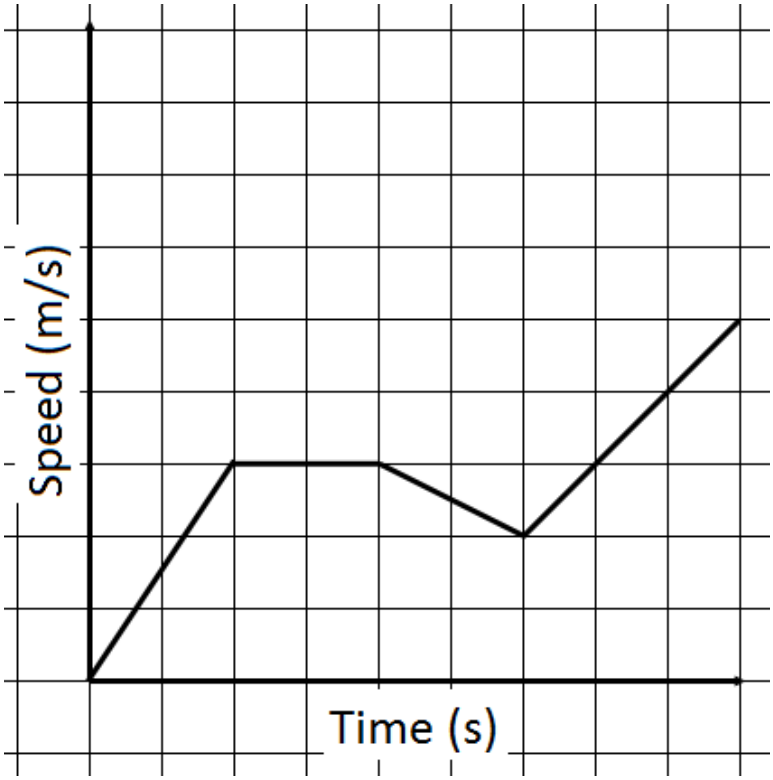
The **area under a speed-time graph gives the distance that an object has travelled** up to that point in time. The area can be calculated by adding up the areas of squares and triangles.

The graph below summarises some of the features of a speed-time graph.



Example 2

A cyclist pedals their bike along a straight path, and changes the bike's speed throughout the journey. The motion of the bike is represented by the speed-time graph below.



Each side of a square represents one unit (second or metres per second).

Using the above graph, answer the following questions:

a) Calculate the distance travelled by the bike after the first two seconds.
b) Identify the interval of time where the bike was travelling at a constant speed.
c) Calculate the acceleration of the bike between the fourth and sixth seconds, and describe what is happening in this interval of time.
d) Identify the interval of time in the recorded motion where the bike is accelerating at the greatest rate. Calculate the rate of acceleration in this interval.
e) Calculate the total distance travelled by the bike.
f) Is the total distance travelled by the bike equal to the magnitude of the displacement of the bike? Justify your answer.
g) Describe the motion of the bike over this time period.

Solution

a) The distance travelled is equal to the area under the graph between zero seconds and two seconds.

$$Distance = Area\ under\ graph$$

$$Distance = \frac{1}{2} \times 2 \times 3 = 3\ m/s$$

b) The bike travels at a constant speed when there is no acceleration. Since the gradient of a speed-time graph is equal to acceleration, the bike will travel at a constant speed when the gradient of the graph is zero.

Therefore, the interval of time where the bike was travelling at constant speed is between the second and fourth seconds.

c) The acceleration of any time interval can be calculated in a speed-time graph by calculating the gradient of the graph over that time interval.

$$Acceleration = Gradient$$

$$Acceleration = \frac{2-3}{2} = -0.5\ m/s^2$$

During the interval of time between the fourth and sixth seconds, the bike has a negative acceleration. Therefore, the bike is slowing down in this interval of time, at a rate of $0.5\ \mathrm{m/s^2}$.

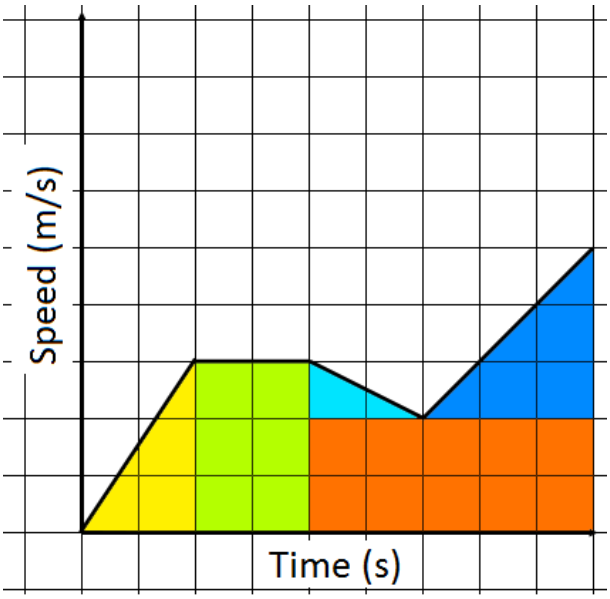
d) Since acceleration equals the gradient of the speed-time graph, the interval of time with the greatest acceleration would have the steepest gradient. This occurs over the first two seconds of motion.

$$Acceleration = Gradient$$

$$Acceleration = \frac{3-0}{2} = 1.5\ m/s^2$$

Therefore, the bike is accelerating at the greatest rate of $1.5\ \mathrm{m/s^2}$ over the first two seconds of motion.

e) The total distance travelled is equal to the total area under the graph for the entire duration of motion. To easily calculate the total area underneath the graph of irregular shapes, it is ideal to divide up the area under the graph into triangles and rectangles. The total area can then be calculated by summing the individual areas of the triangles and rectangles, as demonstrated below.



$$Distance = Area\ under\ graph$$

$$Distance = \left(\frac{1}{2} \times 2 \times 3\right) + (2 \times 3) + \left(\frac{1}{2} \times 1 \times 2\right) + \left(\frac{1}{2} \times 3 \times 3\right) + (2 \times 5)$$

$$Distance = 3 + 6 + 1 + 4.5 + 10 = 24.5m$$

Therefore, the total distance travelled by the bike is 24.5 metres.

f) Recall that displacement measures the distance that an object finishes from its starting point. If an object travels forwards and then travels backwards to its starting point, it has travelled some distance, but its overall displacement is zero.

Since the information in the question states that the cyclist is travelling in a straight line, the magnitude of the displacement should be equal to the distance travelled. If the path was curved, it is unlikely that the magnitude of the displacement would equal the total distance travelled.

g) The bike initially starts from rest and accelerates over two seconds to a speed of 3 m/s. Following this, the bike maintains a steady speed for a further two seconds, before decelerating to a speed of 2 m/s over a period of two seconds. Afterwards, the bike starts accelerating for three seconds to a final speed of 5 m/s. The total distance travelled by the bike during this time is 24.5 m.

TERM 1 – WEEK 4 – HOMEWORK

1. Describe the information that a distance-time graph can provide. **[2 marks]**

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2. Explain what the gradient of a distance-time graph represents, and hence what a flat section and steep section of such a type of graph indicates. **[3 marks]**

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3. From the distance-time graph below, identify the number of times the object is not moving, between time x and y. **[1 mark]**

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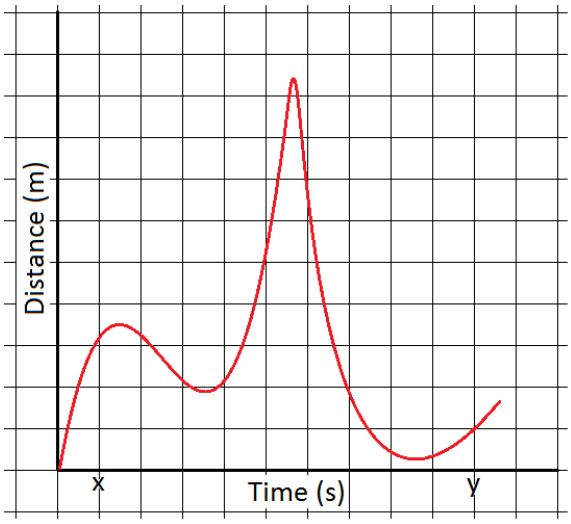

4. Describe the information that a speed-time graph can provide. **[2 marks]**

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5. Explain what the gradient of a speed-time graph represents, and therefore explain what a flat section and steep section of such a type of graph indicates. **[3 marks]**

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6. Explain what the area below a speed-time graph represents. **[2 marks]**

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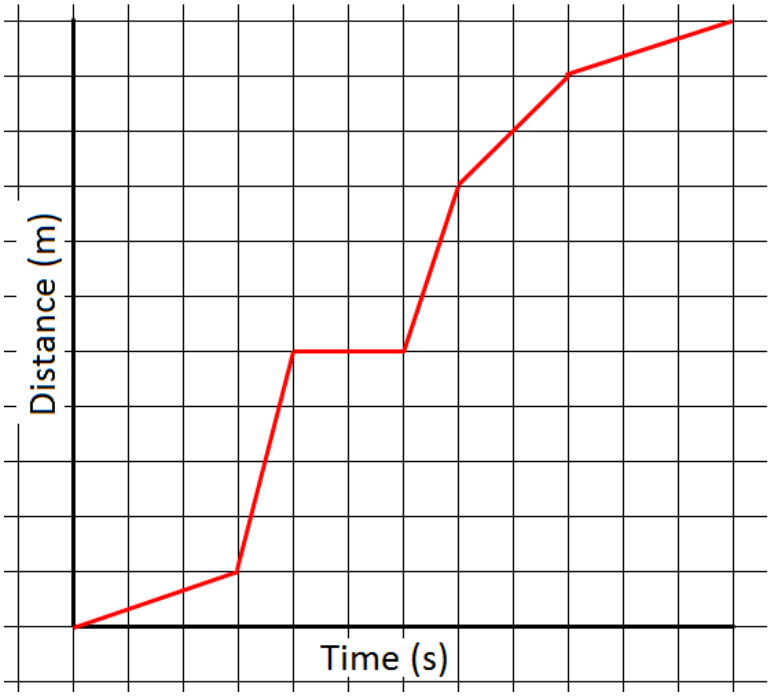
7. The graph below represents the motion of an object that has been recorded. Each side of the squares represent one unit (second or metre).



Using the above graph, answer the following questions:

a. Identify the total distance travelled by the object. **[1 mark]**

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b. During which interval(s) of time was the object stationary? **[1 mark]**

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c. Calculate the average speed of the object in the third second. **[1 mark]**

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d. Calculate the average speed of the entire journey of this object. **[1 mark]**

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e. During which interval of time is the object moving at the greatest speed? Calculate the average speed of the object during this interval of time. **[2 marks]**

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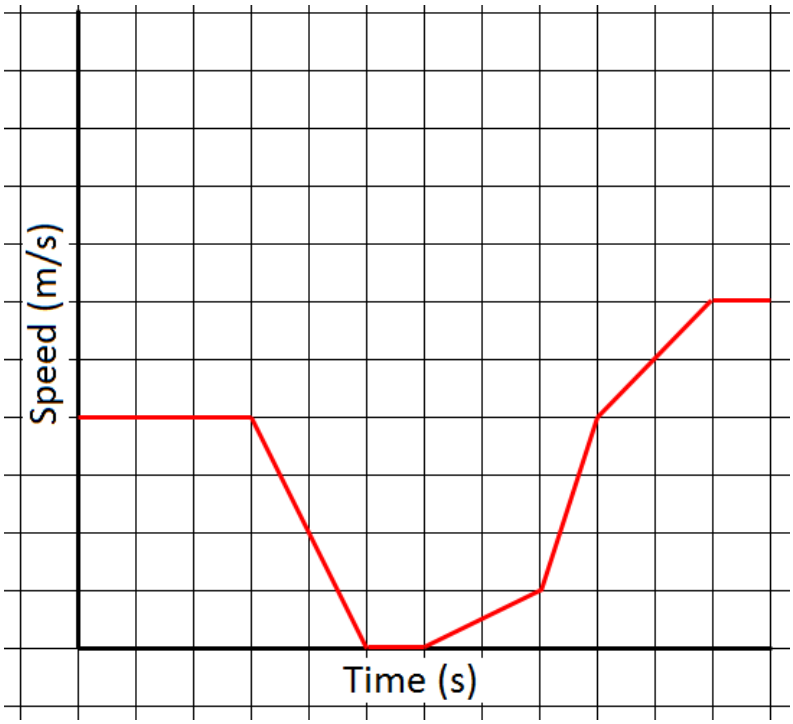
8. The graph below shows the motion of a car. Each side of the squares represents one unit. (second or metres per second)



a. Identify the initial speed the car is travelling at. **[1 mark]**

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b. Identify the total length of time shown, where the car is stationary. **[1 mark]**

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c. Identify the total length of time where the car is travelling at a constant speed. (Hint: 'constant speed' includes cases where speed = 0) **[2 marks]**

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d. Calculate the distance travelled over the first three seconds. **[1 mark]**

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e. Calculate the distance travelled over the entire journey. **[2 marks]**

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f. Calculate the average speed travelled by the car over the entire journey. **[1 mark]**

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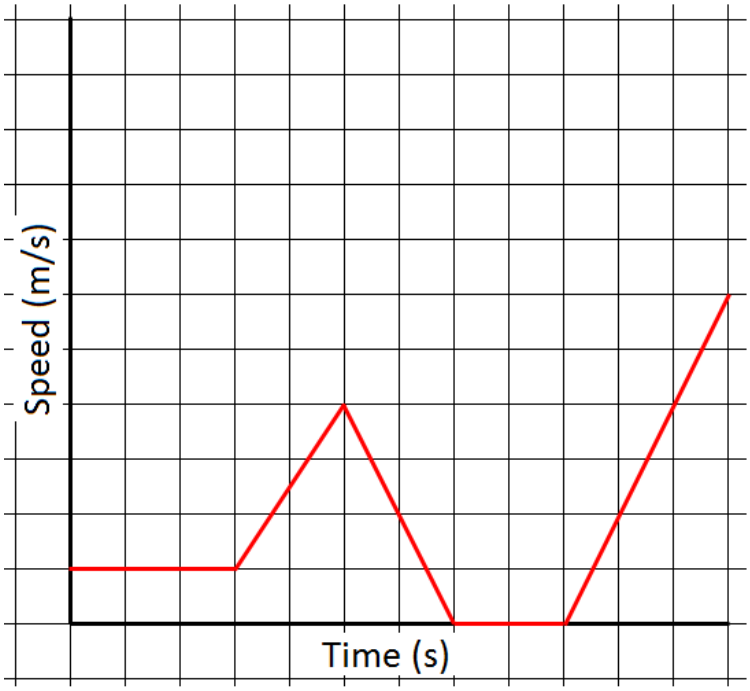
9. An object varies its speed during a journey. This journey is represented in the graph below. Each side of the squares represents one unit. (second or metres per second).



a. Determine the average acceleration of the object from time = 0 to time = 5. **[1 mark]**

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b. Identify the interval of time with the greatest magnitude of acceleration, and calculate the acceleration during this interval of time. **[2 marks]**

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c. Identify the interval of time where the object is stationary. **[1 mark]**

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d. Is the average speed of the object equal to the average velocity of the object? Explain why or why not. **[3 marks]**

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End of homework