Preliminary Mathematics Extension 1

3D Trigonometry

Term 1 – Week 5

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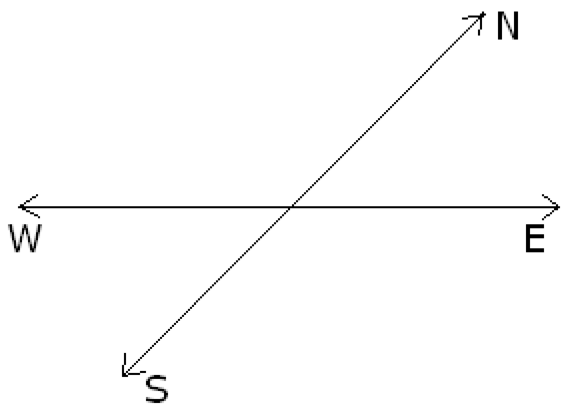
Term 1 – Week 5 – Theory

3 Dimensional Trigonometry:

Problems involving 3-dimensional trigonometry is an extension on the 2U Mathematics trigonometry course. It is important to visualise the situation and draw a reasonably accurate sketch. Tips on how to draw an accurate sketch are listed below:

(i) The 2-dimensional plane should be drawn by using a horizontal line to represent the West-East line and an oblique line to represent the North-South line. An ideal 2-dimensional plane is drawn below:



(ii) Students should understand that in the above diagram the two lines are perpendicular to each other.

(iii) Students should then label other information from the question on the above plane.

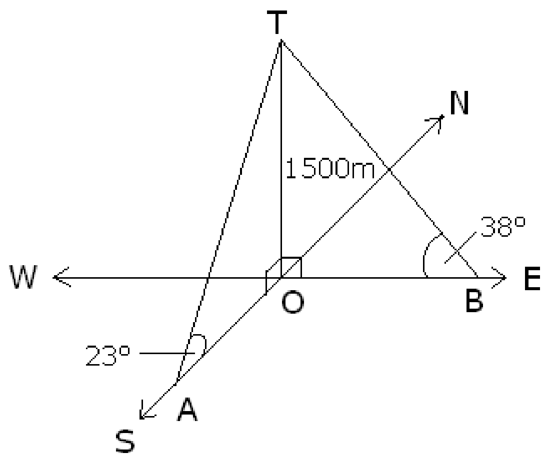
2-Dimensional in a 3-Dimensional Plane:

Example:

The height of a building is measured to be 1500 m. Person A and B are due south and east of the building respectively. The angles of elevation from person A and person B to the top of the building are measured to be 23° and 38° respectively. How far is each person from the base of the building?

Solution:

Let the base of the building be O and the top of the building be T.



$$\tan 23° = \frac{1500}{OA}$$

$$OA = \frac{1500}{\tan 23°}$$

$$\therefore OA \approx 3533.78\, m \text{ (to the nearest 2 decimal places)}$$

$$\tan 38° = \frac{1500}{OB}$$

$$OB = \frac{1500}{\tan 38°}$$

$$\therefore OB \approx 1919.91\, m \text{ (to the nearest 2 decimal places)}$$

∴ Person A and person B are approximately 3533.78 m and 1919.91 m from the base of the building respectively.

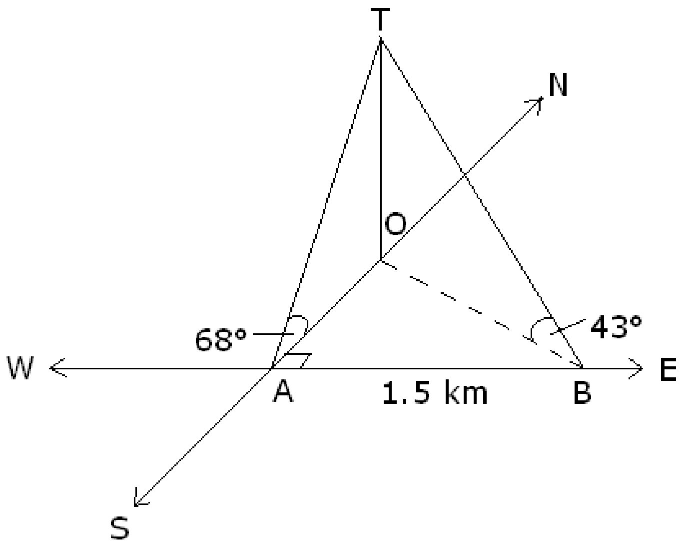
Right-Angled Triangle in the Horizontal Plane:

Example:

A lighthouse is observed from two points A and B, in the same horizontal plane as the base of the lighthouse. Point A is due south of the lighthouse, the angle of elevation to the top of the lighthouse is 68°. Point B is due east of point A, the angle of elevation to the top of the lighthouse is 43°. If the points A and B are 1.5 km apart, find the height of the lighthouse to the nearest 0.1m.

Solution:

Let the base of the lighthouse be O and the top of the lighthouse be T.



In ΔAOT,

$$\tan 68° = \frac{OT}{OA}$$

$$OA = \frac{OT}{\tan 68°}$$

In ΔBOT,

$$\tan 43° = \frac{OT}{OB}$$

$$OB = \frac{OT}{\tan 43°}$$

In ΔABO,

By Pythagoras Theorem, $AB^2 = OB^2 - OA^2$

$$1500^2 = \left(\frac{OT}{\tan 43°}\right)^2 - \left(\frac{OT}{\tan 68°}\right)^2$$

$$1500^2 = \frac{OT^2}{\tan^2 43°} - \frac{OT^2}{\tan^2 68°}$$

$$1500^2 = OT^2\left(\frac{1}{\tan^2 43°} - \frac{1}{\tan^2 68°}\right)$$

$$OT^2 = \frac{1500^2}{\left(\frac{1}{\tan^2 43°} - \frac{1}{\tan^2 68°}\right)}$$

$$OT = \sqrt{\frac{1500^2}{\left(\frac{1}{\tan^2 43°} - \frac{1}{\tan^2 68°}\right)}}$$

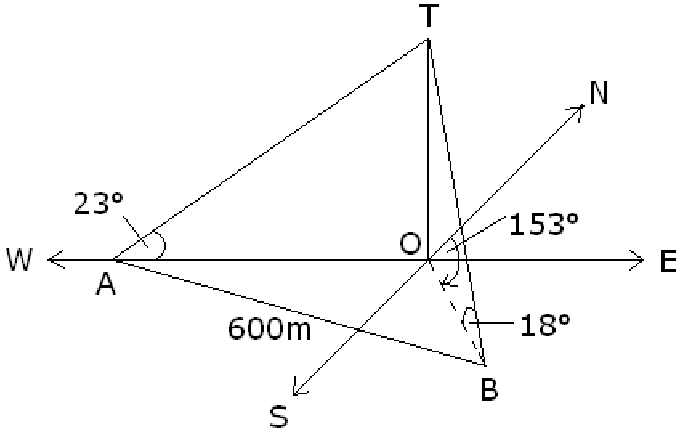
$$\therefore OT = 1510.0\ m$$

∴ The height of the lighthouse is 1510.0 m.

Non-Right-Angled Triangle in the Horizontal Plane:

Example:

The angle of elevation to the top of a building from a point A due west of the building is 23°. Point B has a bearing of $153°T$ from the base of the building and the angle of elevation to the top of the building is 18°. If the points A and B are 600 m apart:



(i) Show that $\angle\text{AOB} = 117°$.
(ii) Show that $\text{AO} = \text{OT}\tan 67°$
(iii) Similarly, show that $\text{BO} = \text{OT}\tan 72°$.
(iv) Hence, evaluate OT to the nearest metre.

Solution:

(i) $\angle\text{AOB} = 270° - 153°$
$\therefore \angle\text{AOB} = 117°$

(ii) $\angle\text{ATO} = 90° - 23°$
$= 67°$
$$\tan 67° = \frac{\text{AO}}{\text{OT}}$$
$\therefore \text{AO} = \text{OT}\tan 67°$

(iii) $\angle\text{BTO} = 90° - 18°$
$\therefore \angle\text{AOB} = 72°$
$$\tan 72° = \frac{\text{BO}}{\text{OT}}$$
$\therefore \text{BO} = \text{OT}\tan 72°$

(iv) In ΔAOB, by Pythagoras' Theorem,

$$600^2 = \mathrm{AO}^2 + \mathrm{BO}^2 - 2(\mathrm{AO})(\mathrm{BO})\cos 117°$$
$$600^2 = (\mathrm{OT}\tan 67°)^2 + (\mathrm{OT}\tan 72°)^2 - 2(\mathrm{OT}\tan 67°)(\mathrm{OT}\tan 72°)\cos 117°$$
$$600^2 = \mathrm{OT}^2\tan^2 67° + \mathrm{OT}^2\tan^2 72° - 2\mathrm{OT}^2\tan 67°\tan 72°\cos 117°$$
$$600^2 = \mathrm{OT}^2(\tan^2 67° + \tan^2 72° - 2\tan 67°\tan 72°\cos 117°)$$
$$\mathrm{OT}^2 = \frac{600^2}{\tan^2 67° + \tan^2 72° - 2\tan 67°\tan 72°\cos 117°}$$
$$\mathrm{OT} = \sqrt{\frac{600^2}{\tan^2 67° + \tan^2 72° - 2\tan 67°\tan 72°\cos 117°}}$$
$$\mathrm{OT} = \frac{600}{\sqrt{\tan^2 67° + \tan^2 72° - 2\tan 67°\tan 72°\cos 117°}}$$
$$\therefore \mathrm{OT} \approx 129\text{ m (to the nearest metre)}$$

Term 1 – Week 5 – Homework

3 Dimensional Trigonometry:

1. P and Q are points on the same plane as the base of a tower with height 300 metres. The angles of elevation from P and Q to the top of the tower are 45° and 60° respectively. Find the distance:
   a) From P to the base of the tower.
   b) From Q to the base of the tower.

2. OT is a flagpole where O is the base of the pole and T is the top of the pole. The height of OT is 50 metres. From points A and B on the same plane as O, the angles of elevation of T are 36° and 40° respectively.
   a) Illustrate the above information in a diagram.
   b) Find the distance from A to O.
   c) Find the distance from B to O.
   d) If the A and B are 100 metres apart, find the magnitude of the angle AOB.

3. A tower is observed from two points, A and B. From A, due south of the tower, the angle of elevation of the top of the tower is 35° and from B, due east of the tower, the angle of elevation of the top of the tower is 25°. If A and B are 2.5km apart, find the height of the tower to the nearest metre.

4. From P, due south of a hill, the angle of elevation of the top of the hill is 40° and from Q, due east of P, the angle of elevation of the top of the hill is 20°. If the distance from P to Q is 2000 metres, find:
   a) The height of the hill.
   b) The distance from P to the base of the hill
   c) The distance from Q to the base of the hill.
   (Correct all your answers to the nearest 0.1m).

5. A vertical tower is observed from two points, P and Q, on the same plane as the base of the tower. From P, due north of the tower, the angle of elevation is 60° and from Q, due west of the tower, the angle of elevation is 45°. If P and Q are 150 metres apart, find:
   a) The height of the tower.
   b) The distance from P to the base of the tower.
   c) The distance from Q to the base of the tower.

6. The angle of elevation of a tower BT from a point P due east of it is 15°. From another point Q, the bearing of the tower is 38°T and the angle of elevation is 10°. The two points P and Q are 800 metres apart and on the same plane as the base of the tower, B.
   a) Show that the angle PBQ is 128°.
   b) Find an expression for QB in terms of h, the height of the tower.
   c) Find an expression for PB in terms of h, the height of the tower.
   d) Find the height of the tower, h to the nearest 0.1 metre. (*Hint:* Use the cosine rule).

7. OT is a tower with height h. The base and top of the tower are represented by O and T respectively. From a point P, due west of the tower, the angle of elevation of T is 8° and from a point Q, the bearing of the tower is 314°T and the angle of elevation of T is 16°. If the two points P and Q are 1.2km apart and on the same plane as the base of the tower, O.
   a) Show that the magnitude of angle POQ is 136°.
   b) By considering the triangle QOT, show that $QO = h\tan 74°$.
   c) Similarly, show that $PO = h\tan 82°$.
   d) Find h, the height of the tower to the nearest metre.

8. P and Q are the base of two towers each with height 40 metres. Q is due north of P. The two towers are observed from a point A on the same plane as the base of the towers and due west of P. The angles of elevation from A to the tops of the towers P and Q are 60° and 40° respectively. Find:
   a) The distance from A to the base of each tower.
   b) The distance between the two towers.

End of homework