HSC Mathematics Extension 1

Probability

Term 2 – Week 3

Student Name: ……………………………….

Class code: …………………………………..

Teacher name: ……………………………..

DUX College

T: (02) 8007 6824
E: info@duxcollege.com.au
W: www.duxcollege.com.au

Term 2 – Week 3 – Theory

Independent Events and Dependent Events for Small and Large Populations:

Dependent Events:

For dependent events, the outcome of an event will depend on the outcome of other events.

For example, if names are drawn out of a hat without replacement, the probability of drawing out a particular name on the second draw will depend on what was drawn out in the first draw. Using a probability tree is usually the best method for questions about sampling without replacement.

Hint: If you see the words 'without replacement', you will be dealing with dependent successive events. If you see the words 'with replacement', you will be dealing with independent successive events.

Mathematically, if A and B are two dependent events, then

$$P(A \text{ and } B) = P(A) \times P(B, \text{given that A has occurred})$$

This is usually notated as

$$P(A \cap B) = P(A) \times P(B|A)$$

Example:

15 cards, numbered from 1-15, are placed in a hat. Two cards are drawn out without replacement. Find the probability that:

(i) the first card is odd
(ii) the second card is odd
(iii) at least one of the cards is odd

Solution:

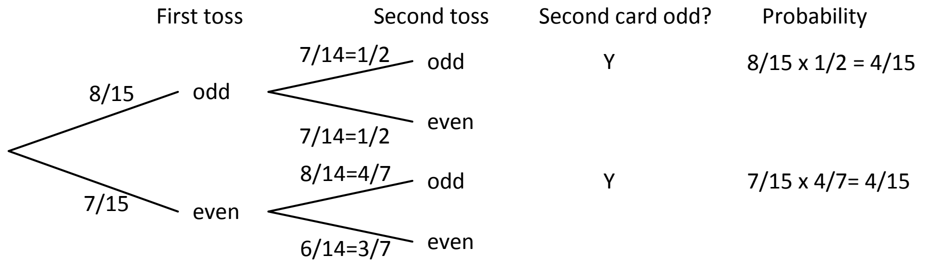
(i)

Since there are 8 odd numbers and 7 even numbers in the hat,

$\therefore P(1^{\text{st}} \text{ odd}) = \dfrac{8}{15}$

(ii)

© Dux College 2010 | Do not distribute



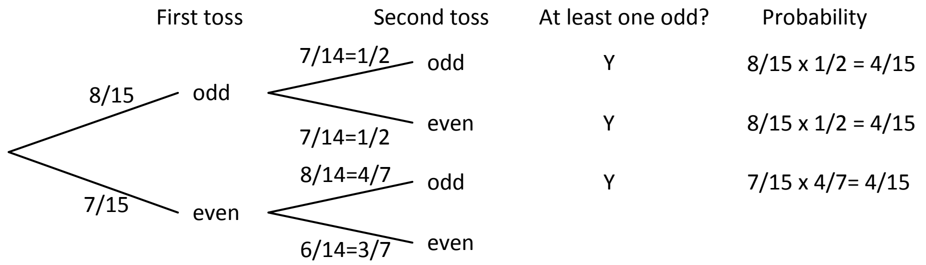
$$\therefore P\left(2^{\text{nd}}\text{ odd}\right) = \frac{4}{15} + \frac{4}{15}$$
$$= \frac{8}{15}$$

(iii)

Method 1:

$$P(\text{at least one odd}) = 1 - P(\text{both even})$$
$$= 1 - \frac{7}{15} \times \frac{6}{14}$$
$$= 1 - \frac{1}{5}$$
$$= \frac{4}{5}$$

Method 2: probability tree diagram



$$\therefore P(\text{at least one odd}) = \frac{4}{15} + \frac{4}{15} + \frac{4}{15}$$
$$= \frac{12}{15}$$
$$= \frac{4}{5}$$

Hint: Always go for the method that requires the least counting. In this case, method 1 is preferable.

© Dux College 2010 | Do not distribute

Sampling from a Small Population:

When sampling from a small population, it is usually helpful to determine whether events are dependent or independent, as this can affect the result obtained.

Example:

A card is drawn at random from a standard deck, noted, and shuffled back in. A second card is then drawn from the deck again. Find the probability that:

(i) the first card is a heart.
(ii) the second card is a heart.
(iii) both cards are hearts.

Solution:

(i)

$$\therefore P(1^{\text{st}} \text{ is heart}) = \frac{1}{4}$$

(ii)

$$\therefore P\left(2^{\text{nd}} \text{ is heart}\right) = \frac{1}{4}$$

(iii)

$$\begin{aligned}\therefore P\left(1^{\text{st}} \text{ is heart and } 2^{\text{nd}} \text{ is heart}\right) &= \frac{1}{4} \times \frac{1}{4} \\ &= \frac{1}{16}\end{aligned}$$

© Dux College 2010 | Do not distribute

Example:

Two cards are drawn at random from a standard deck, without replacement. Find the probability that:

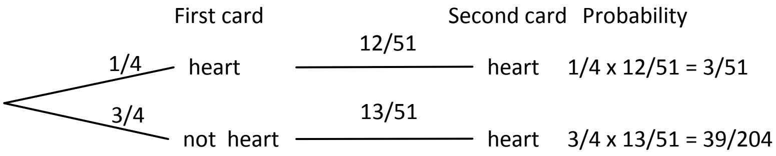
(i) the first card is a heart
(ii) the second card is a heart
(iii) both cards are hearts

Solution:

(i)

$$\therefore P(1^{\text{st}} \text{ is heart}) = \frac{1}{4}$$

(ii)



Hint: In this example, only the branches which lead to the second card being a heart have been included. This can save a lot of time and make your probability tree diagram look less cluttered.

$$\begin{aligned}\therefore P\left(2^{\text{nd}} \text{ is heart}\right) &= \frac{3}{51} + \frac{39}{204} \\ &= \frac{51}{204} \\ &= \frac{1}{4}\end{aligned}$$

(iii)

<u>Method 1:</u>

From the diagram for part (ii),

$$\therefore P(\text{both are hearts}) = \frac{3}{51}$$

<u>Method 2:</u>

Since the two events are dependent,

$$\begin{aligned}\therefore P\left(1^{\text{st}} \text{ is heart and } 2^{\text{nd}} \text{ is heart}\right) &= P(1^{\text{st}} \text{ is heart}) \times P\left(2^{\text{nd}} \text{ is heart, given } 1^{\text{st}} \text{ is heart}\right) \\ &= \frac{1}{4} \times \frac{12}{51} \\ &= \frac{3}{51}\end{aligned}$$

© Dux College 2010 | Do not distribute

Example:

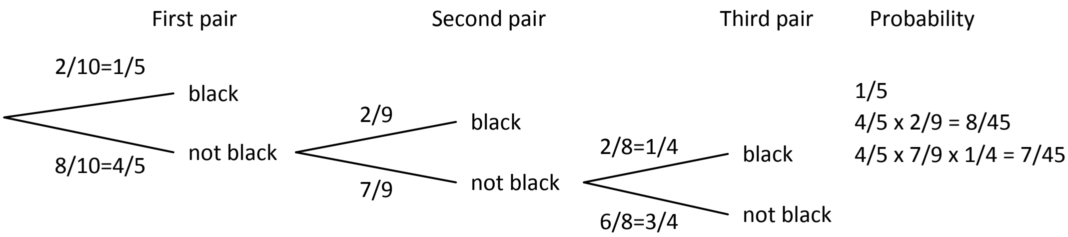
Amy has 10 pairs of socks in her sock drawer, two pairs of which are black. If she randomly draws out three pairs of socks, without replacement, find the probability that:

(i) at least one pair is black.
(ii) all three pairs are black.

Solution:

(i)

Method 1: probability tree diagram



Once we draw out a black pair, the criteria 'at least one pair is black' is fulfilled. Therefore we need not consider where the tree branches beyond that point.

$$\begin{aligned}\therefore P(\geq 1 \text{ black}) &= \frac{1}{5} + \frac{8}{45} + \frac{7}{45} \\ &= \frac{24}{45} \\ &= \frac{8}{15}\end{aligned}$$

Method 2:

$$\begin{aligned}\therefore P(\geq 1 \text{ black}) &= 1 - P(0 \text{ black}) \\ &= 1 - \frac{8}{10} \times \frac{7}{9} \times \frac{6}{8} \\ &= 1 - \frac{7}{15} \\ &= \frac{8}{15}\end{aligned}$$

(ii) Since only 2 pairs are black,

$\therefore P(3 \text{ black}) = 0$

© Dux College 2010 | Do not distribute

Sampling from a Large Population:

When taking small samples from a large population, the effects of previous events are less significant.

For example, if we draw two marbles without replacement out of a bag of 10 marbles, 20% or which are black, the probability that the first one we draw out is black is given by $20\% = 2/10$. The probability that the second one is also black, given that the first one is black, is given by $1/9 = 0.\dot{1}$. However, if we draw two marbles out of a bag of 1000 marbles, 20% of which are black, the probability that the first one we draw out is black is given by $20\% = 200/1000 = 1/5$ and the probability that the second one is also black is given by $199/1999 \approx 20\%$. This is approximately equal to if we had replaced the first black marble.

Because of this approximation, it is generally safe to assume when taking small samples from a large population that removing the sample does not affect the percentage composition of the population.

Example:

In a factory, 10% of the products are defective. If 3 products are taken as a random sample, find the probability that:

(i) none are defective.
(ii) all of them are defective.
(iii) more are defective than not defective.

Solution:

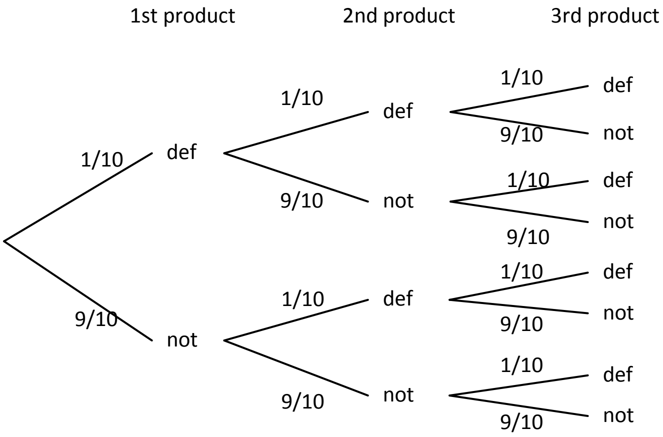
(i)

$$\begin{aligned}\therefore P(0\text{ defective}) &= \frac{9}{10}\times\frac{9}{10}\times\frac{9}{10}\\ &= \frac{729}{1000}\end{aligned}$$

(ii)

$$\begin{aligned}\therefore P(\text{all defective}) &= \frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\\ &= \frac{1}{1000}\end{aligned}$$

(iii)



More defective	Probability
Y	1/10 x 1/10 x 1/10 = 1/1000
Y	1/10 x 1/10 x 9/10 = 9/1000
Y	1/10 x 9/10 x 9/10 = 81/1000
Y	9/10 x 1/10 x 1/10 = 9/1000

$$\therefore P(\text{more defective}) = \frac{1}{1000} + \frac{9}{1000} + \frac{81}{1000} + \frac{9}{1000}$$
$$= \frac{100}{1000}$$
$$= \frac{1}{10}$$

© Dux College 2010 | Do not distribute

Term 2 – Week 3 – Homework

Independent and Dependent Events:

1. A coin is flipped and a die is rolled.
   a) Are these two events dependent or independent?
   b) Find the probability that:
      i. The coin lands on 'heads'
      ii. The die lands on an even number
      iii. The coin lands on 'heads' and the die does not land on a 4

2. Six marbles are placed in a bag. Two are black and four are white. Two marbles are drawn with replacement.
   a) Are these two events dependent or independent?
   b) Find the probability that:
      i. The first marble is black
      ii. Both marbles are black
      iii. There is exactly one white marble

3. Alice, Betty, Chris, Doug and Emily place their names in a hat. Two names are drawn without replacement.
   a) Are these two events dependent or independent?
   b) Find the probability that:
      i. Alice is drawn first
      ii. Alice is drawn out at some stage
      iii. Neither of the boys are chosen

4. A box contains 4 blue balls and 6 white balls. Two balls are drawn without replacement.
   a) Are these two events dependent or independent?
   b) Find the probability that both balls are the same colour

5. Ten cards, numbered from 1-10, are shuffled together. Two cards are drawn without replacement.
   a) Are these two events dependent or independent?
   b) Find the probability that:
      i. The first is a 10
      ii. The second is even
      iii. The first is a 10 and the second is even
      iv. Both cards are odd
      v. Neither card is prime

6. There are 12 beads in a bag. 3 are white, 4 are green and 5 are blue. Two beads are selected at random. Find the probability that:
   a) They are both white
   b) They are both green
   c) Neither is blue

© Dux College 2010 | Do not distribute

7. In a class of 30 schoolchildren, 18 are female and 12 are male. 11 of the females wear braces and 2 of the males wear braces. If one female and one male are chosen at random, find the probability that:
   a) Both wear braces
   b) Only the female wears braces
   c) Only one of them wears braces

8. There are 20 items in a box, 4 of which are defective. If a sample of three is chosen at random, find the probability that:
   a) Exactly one is defective
   b) At most one is defective

9. A die is thrown and a card is drawn from a standard deck of 52 cards. Find the probability that:
   a) The number is even and the card is a heart
   b) The number is prime and the card is not the same number

10. Bob takes three subjects: Mathematics, English and Chemistry. The probability that passes Mathematics is 9/10, the probability that he passes English is 4/5 and the probability that he passes Chemistry is 5/7. Find the probability that Bob:
    a) Fails exactly one subject
    b) Passes all three subjects
    c) Passes more subjects than he fails

11. Two patients participate in clinical trials. Patient A trials a drug that has a 20% success rate. Patient B trials a drug that has a 40% success rate. Find the probability that:
    a) Only patient A is cured
    b) Both patients are cured
    c) Neither patient is cured

12. 100 tickets are sold for an Easter raffle. If there are three prizes, find the probability that a person who buys 5 tickets wins:
    a) No prizes
    b) All three prizes
    c) Only first prize
    d) Only second and third prize

13. There are 15 light globes in a box. 2 are defective. If two light globes are chosen at random, find the probability that:
    a) None are defective
    b) Both are defective

14. Class A consists of 12 girls and 18 boys. Class B consists of 17 girls and 13 boys. A class captain is chosen at random for each class. Find the probability that:
    a) The captain of Class A is a boy
    b) Both class captains are girls
    c) Exactly one class captain is a girl

15. At a concert, 60% of people are male. If a sample of two is selected at random, what is the probability that both are male?

© Dux College 2010 | Do not distribute

16. Two cards are drawn without replacement from a standard deck of 52 cards. Find the probability that:
    a) Neither card is a club
    b) The first is an ace and the second is a queen
    c) The first is an ace and the second is a heart
    d) The sum of the cards is 12 (assuming that J, Q and K are worth 10 points each and ace is worth 1)

17. Three cards are drawn at random from a standard deck of 52 cards. Find the probability that:
    a) They are all of the same suit
    b) They are all of different suits

18. A loaded die is thrown. It is known that P(6)=1/3, P(1)=1/24 and each of the other numbers is equally likely. If the die is tossed twice, find the probability that:
    a) Both numbers are 6's
    b) A double is thrown
    c) Two odd numbers are thrown

19. In a box of chocolates, there are 10 dark chocolates, 15 milk chocolates and 5 white chocolates. If two chocolates are selected at random, find the probability that:
    a) Neither is a white chocolate
    b) Both are milk chocolates
    c) The two are not of the same type of chocolate

20. A die is tossed three times and the number on the uppermost face noted. Find the probability that:
    a) Three 6's are thrown
    b) A triple is thrown
    c) 3 odd numbers are thrown
    d) An odd number, an even number and a 6 are thrown, in that order?

End of Homework

© Dux College 2010 | Do not distribute