HSC Physics

Space

Week 7

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Week 7 – Theory

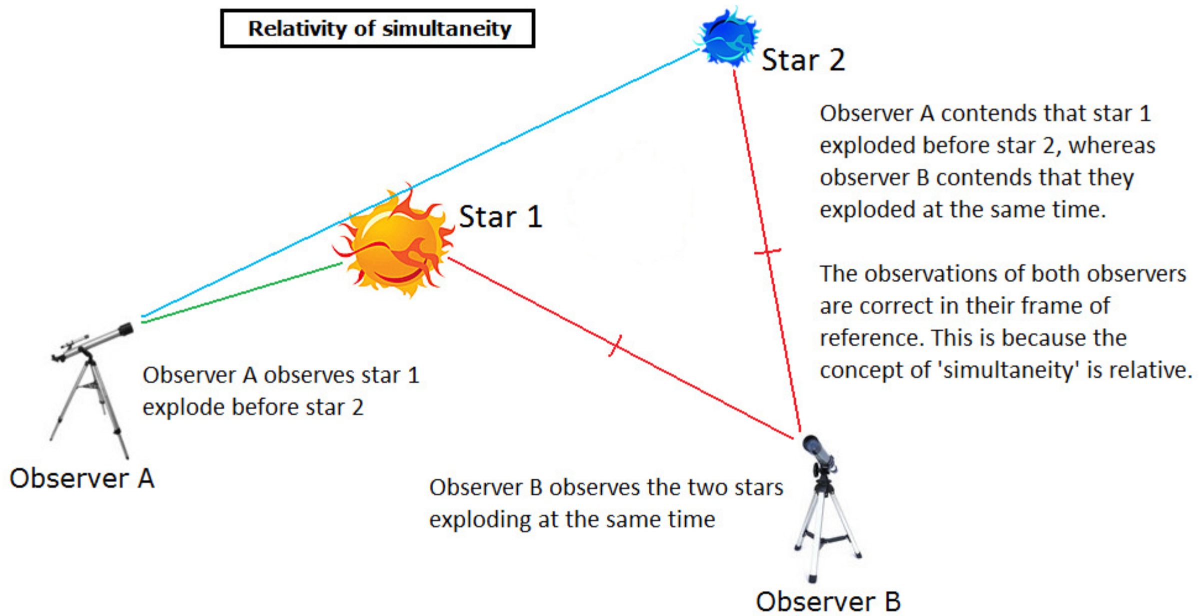
- **Explain qualitatively and quantitatively the consequence of special relativity in relation to: relativity of simultaneity, the equivalence between mass and energy, length contraction, time dilation and mass dilation**
- **Solve problems and analyse information using:**

$$E = mc^2 \qquad l_v = l_0\sqrt{1-\frac{v^2}{c^2}} \qquad t_v = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad = m_v = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Relativity of simultaneity

Simultaneity is the notion of two different events occurring at the exact same time. The relativity of simultaneity refers to Einstein's idea that: *events judged to be simultaneous by an observer in one frame of reference, may not be perceived to be simultaneous by an observer in relative motion to the first observer*.

This can be seen in the following thought experiment.



Consider two stars exploding in distant space. Observer B is equidistant to both explosions, so the light from both events reaches B simultaneously. Both events therefore can be considered to have occurred simultaneously relative to B.

However, Observer A would see the orange star explode before the blue star, as the orange star is much closer to A than the blue star. Relative to Observer A, the events are not simultaneous. **Both observations are correct**, that is it is impossible to say in an absolute sense whether two events occur at the same time, if those events are separated in space.

Time dilation

When doing calculation questions, remember the **definition of the rest frame – the frame which the event in question is in**. This is important, since the formula for time dilation (as well as mass dilation and length contraction) is dependent on this distinction.

The time taken for an event to occur within its rest frame is called the proper time (t_0). Observers in different reference frames in relative motion will always judge the time taken to be longer (t_v).

$$t_v = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where:

t_0 is the time taken in the rest frame (s)

t_v is the time observed from any reference frame in relative motion to the rest frame (s)

v is the velocity of the frame of reference relative to the rest frame (ms^{-1})

c is the speed of light ($3 \times 10^8 ms^{-1}$)

In the most common type questions involving spacecraft / vehicles moving at relativistic speeds, t_0 is the time experienced inside the spacecraft, and t_v is the time experienced outside of the spacecraft. However this is just a generalisation – it is important to understand the definition of which is the rest frame.

Example 1

A train travelling at 0.4*c* passes through a station. A passenger onboard the train waves for exactly 1.200 seconds at a friend standing on the platform. Calculate how long the friend sees the passenger waving.

Solution

$$t_v = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The passenger inside the train is the rest-frame, because the passenger is in the same frame as the event (hand wave). 1.2 seconds experienced by the passenger therefore means $t_0 = 1.200$. The time experienced by the friend on the platform is t_v. Therefore:

$$t_v = \frac{1.200}{\sqrt{1 - \left(\frac{0.4c}{c}\right)^2}} = 1.309 \text{ seconds}$$

Example 2

A spaceship flies past Earth at 0.7c, and observes an explosion occur on the surface of the Earth, lasting 3.2 seconds. How long does this explosion take, relative to a bystander on Earth?

Solution

This time the rest frame is on Earth, because the question is asking how long an event takes relative to an observer in the same frame as the event.

$$t_v = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$3.2 = \frac{t_0}{\sqrt{1 - (0.7)^2}}$$

$$t_0 = 2.3s$$

That is, the explosion actually takes 2.3s on Earth. This is because although we think of the spaceship moving at 0.7c, the Earth is actually moving relative to the spaceship at 0.7c. The only difference between this question and Example 1 is that the event in question (the explosion) happened on Earth, whereas in Example 1, the event in question (the hand wave) happened on the train.

Example 3

A pion must survive for at least 1 microsecond for a scientific experiment to work. The pion's average lifetime is $2.60 \times 10^{-8} s$ in its rest frame. How fast must the pion travel in order to allow the experiment to work?

Solution

We are given the lifetime of the particle in its rest frame. Therefore:

$$t_v = 1 \times 10^{-6} = \frac{2.60 \times 10^{-8}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Making $\frac{v}{c}$ the subject:

$$1 - \frac{v^2}{c^2} = \left(\frac{2.60 \times 10^{-8}}{1 \times 10^{-6}}\right)^2$$

$$\frac{v}{c} = \sqrt{1 - \left(\frac{2.60 \times 10^{-8}}{1 \times 10^{-6}}\right)^2}$$

$$= 0.987$$

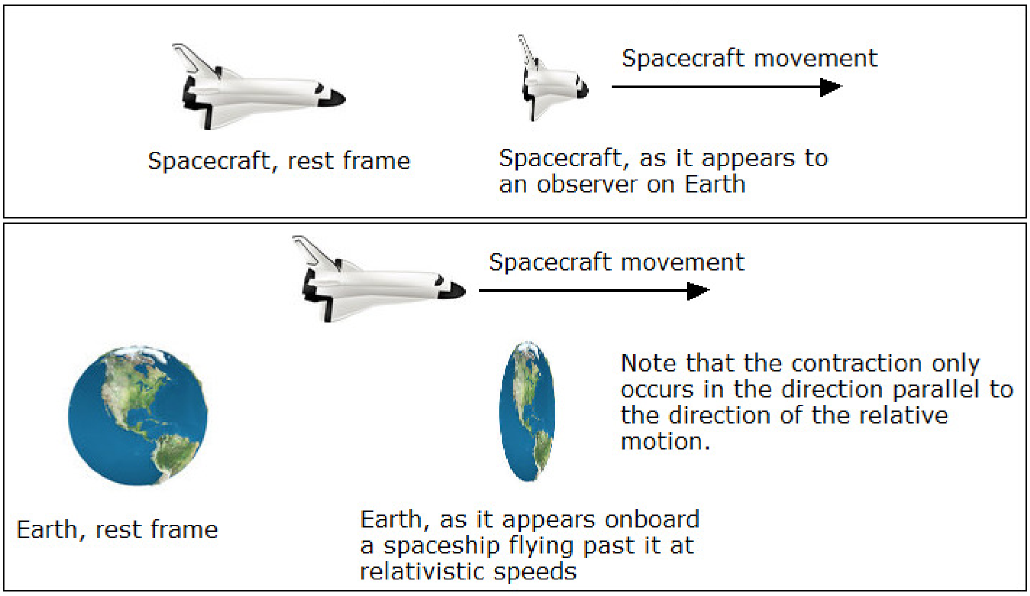
Therefore the pion must be accelerated to a speed of 0.987c in order for the experiment to work.

Length contraction

The length of an object changes depending on its speed relative to observers. For example, as a spacecraft zooms past Earth, observers on Earth will perceive the spacecraft as shorter than its actual length. Similarly, the observer inside the moving spacecraft will see that the diameter of Earth is smaller than its real size. This is known as length contraction and it is a direct result of time dilation.

The greater the velocity, the great the length contraction. It is important to remember that length contraction **only occurs in the direction of the speed**, i.e. the height and width of the moving spacecraft does not change from the Earth's perspective, nor does the height and width of the Earth (just its thickness in the direction of the spacecraft's speed).

Length contraction



Length contraction can be calculated using:

$$l_v = l_0\sqrt{1-\frac{v^2}{c^2}}$$

Where:

- l_0 is the length of the object in the rest frame (m)
- l_v is the length of the object observed from a different frame in relative motion (m)
- v is the velocity of the frame of reference relative to the rest frame (ms^{-1})
- c is the speed of light ($3 \times 10^8 ms^{-1}$)

Example 4

A spaceship of length 230 m zooms past the Earth at 0.7*c*. Calculate the apparent length of the spaceship as seen from Earth.

Solution

$$l_v = l_0\sqrt{1-\frac{v^2}{c^2}}$$

$$l_v = 230 \times \sqrt{1-\frac{(0.7c)^2}{c^2}} = 164 \text{ m}$$

Example 5

Scientists recently discovered an alien spacecraft that can reach amazing speeds of up to 0.9999c. The government wishes to use it to plan an exploration mission to the star Betelgeuse, which is 700 light years away.

(a) What is the length of the journey as observed by the astronauts onboard the spacecraft?
(b) Is this mission feasible, even with relativistic effects?

Solution

(a) With length contraction, the space ahead of the spacecraft contracts in the direction it is heading. This shortens the journey distance relative to those onboard the spacecraft.

$$l_v = 700\sqrt{1-0.9999^2}$$
$$\therefore l_v = 9.899 \; light\ years$$

(b) Without using the time dilation equation, we can work out how long the journey takes the astronauts:

$$t_0 = \frac{l_v}{v}$$
$$= \frac{9.899}{0.9999}$$
$$= 9.900 \; years$$

However, even though the journey is achievable within a lifetime for the astronauts, on Earth, scientists still need to wait slightly over 700 years for the spacecraft to reach the destination, as well as a further 700 years before any information can be sent back. This is not feasible to the scientists on Earth, but the astronauts can reach their destination in well under 700 years.

Mass dilation

The mass of an object increases when it begins to move. The mass of the moving object is greater than that when it is stationary. The greater the velocity of the moving object, the greater its mass will become. For example, as a spacecraft zooms past Earth, observers on Earth will observe the spacecraft to be heavier than its rest mass. Observers aboard the spacecraft measure the spacecraft at its normal rest mass, but every object on Earth, as well as the Earth itself will be heavier relative to those on the spacecraft.

Mass dilation of a moving object can be determined using:

$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where:

m_0 is the rest mass (the mass measured in the rest frame of the object) (kg)

m_v is the mass of an object as observed from any other frame relative to the rest frame (kg)

v is the velocity of the frame of reference relative to the rest frame (ms^{-1})

c is the speed of light ($3 \times 10^8 ms^{-1}$)

According to special relativity, the speed of light is the highest speed that can be reached. As the speed of an object increases, its mass also increases. An applied force is needed to accelerate the object to greater velocities, and this results in a further increase in mass. Subsequent acceleration would require even greater force. As the object approaches the speed of light, its **mass approaches infinity** and hence an infinite force would be required to accelerate it past the speed of light. Therefore, a speed greater than that of light is not attainable.

Example 6

The rest mass of a carbon atom is 1.9932×10^{-26} kg. Determine its mass if it is moving at a quarter the speed of light.

Solution

$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m_v = \frac{1.9932 \times 10^{-26}}{\sqrt{1 - \frac{(0.25c)^2}{c^2}}} = 2.0586 \times 10^{-26} \text{ kg}$$

Example 7

A truck weighing 5,000kg is travelling at a speed of 100km/h. How fast must a 60g tennis ball be moving in order to have the same momentum as the truck?

Solution

First we determine the momentum of the truck:

$$p_{truck} = 5000 \times 100 \times \frac{3,600}{1,000}$$
$$= 138,888.89kgms^{-1}$$

Next we determine the relativistic momentum of the tennis ball:

$$p_{ball} = m_v v = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$= \frac{0.06v}{\sqrt{1 - \frac{v^2}{c^2}}} = 138,888.89$$

We will now square both sides and continue to simplify, finally solving for v:

$$0.036v^2 = \left(1 - \frac{v^2}{c^2}\right) \times 138,888.89^2$$
$$0.036v^2 + v^2\left(\frac{138,888.89}{c}\right)^2 = 138,888.89^2$$
$$v^2 = \frac{138,888.89^2}{0.036 + \left(\frac{138,888.89}{c}\right)^2}$$
$$v = 727,346.79ms^{-1}$$

At this speed, even a humble 60g tennis ball has the same momentum as a truck travelling at 100km/h.

Equivalence between mass and energy

A consequence of special relativity is the principle of equivalence between mass and energy. This means that mass can be converted to energy and vice versa. The conversion is given by Einstein's famous equation$E = mc^2$. Note that because the square of speed of light is an extremely large number, even a tiny amount of mass would yield an enormous amount of energy. That's why atomic bombs are so powerful, since a tiny amount of matter is converted into a large amount of energy.

Energy and mass are related by:

$$E = mc^2$$

Where:

E is the rest energy (J)

m is the mass of the object (kg)

c is the speed of light ($3 \times 10^8 ms^{-1}$)

Example 8

Determine the rest energy of a carbon atom of mass 12.0000 amu (One atomic mass unit is: $1.9932 \times 10^{-26} kg$).

Solution

$$E = mc^2$$
$$= 12 \times 1.9932 \times 10^{-26} \times (3 \times 10^8)^2$$
$$= 1.79 \times 10^{-9} J$$

Example 9

The entire world's energy usage was 474 exojoules ($4.74 \times 10^{20} J$) in 2008. If scientists developed a power plant that could generate power by converting mass into power, how much mass would have been needed to power the entire world in 2008?

Solution

$$E = mc^2$$
$$4.74 \times 10^{20} = mc^2$$
$$m = 4.74 \times \frac{10^{20}}{(3 \times 10^8)^2}$$
$$= 5{,}266.67 kg$$

Week 7 – Homework

- **<u>Explain qualitatively and quantitatively the consequence of special relativity in relation to: relativity of simultaneity, the equivalence between mass and energy, length contraction, time dilation and mass dilation</u>**
- **<u>Solve problems and analyse information using:</u>**

$$E = mc^2 \qquad l_v = l_0\sqrt{1-\frac{v^2}{c^2}} \qquad t_v = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad = m_v = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

1. Explain qualitatively the consequence of special relativity in relation to simultaneity. **[2 marks]**

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2. Explain qualitatively the consequence of special relativity in relation to time dilation. **[2 marks]**

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3. An astronaut travelling at 0.3 *c* takes 6 years on the spaceship to reach her destination. Calculate how much time has passed on Earth. **[2 marks]**

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4. A train travelling at 0.6*c* passes through a station. Jim who is standing on the platform laughs for a while. Passengers on the train observe that Jim laughs for 3.550 seconds. Calculate how long Jim laughs for in his own perspective. **[2 marks]**

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5. Explain qualitatively the consequence of special relativity in relation to length contraction. **[2 marks]**

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6. A train is travelling at three-quarters the speed of light. Each carriage is 15 m long when the train is stationary. Calculate how long each carriage would appear to be to a person outside the moving train. **[2 marks]**

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7. A spaceship flying past Earth at a uniform speed appears to have a length which is 75% of its stationary length. Determine the speed of the spaceship. **[2 marks]**

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8. Explain qualitatively the consequence of special relativity in relation to mass dilation. **[2 marks]**

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9. An electron has a mass of 9.109×10^{-31} kg. Calculate the mass of an electron if it was travelling at 95.9% of the speed of light. **[2 marks]**

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10. If a spaceship is travelling at 0.25 *c* past a planet, calculate the observed mass of the spaceship if its rest mass is 6.5×10^4 kg. **[2 marks]**

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11. Explain qualitatively the consequence of special relativity in relation to equivalence of mass and energy. **[3 marks]**

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12. Calculate the energy equivalent of an electron with a mass of 9.109×10^{-31} kg. **[2 marks]**

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13. An atom of uranium has a rest mass of 3.953×10^{-25} kg. Find its rest energy. **[2 marks]**

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14. As a spaceship flies past Earth, people on Earth observe that the spaceship's length shrinks to 40% of its actual length. Calculate: **[1 mark each]**
    a) the speed of the spaceship relative to Earth

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    b) the observed mass of the spaceship if its rest mass is 1.4×10^5 kg

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    c) the amount of time passed on Earth when one hour has passed on the spaceship

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15. [2002 HSC] In one of Einstein's famous thought experiments, a passenger travels on a train that passes through a station at 60% of the speed of light. According to the passenger, the length of the train carriage is 22 m from front to rear.
    a) A light in the train carriage is switched on. Compare the velocity of the light beam as seen by the passenger on the train and a rail worker standing on the station platform. **[1 mark]**

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    b) Calculate the length of the carriage as observed by the rail worker on the station platform. **[3 marks]**

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16. [2002 HSC] A spaceship is travelling at a very high speed. What effects would be noted by a stationary observer? **[1 mark]**

a) Time runs slower on the spaceship and it contracts in length

b) Time runs faster on the spaceship and it contracts in length

c) Time runs slower on the spaceship and it increases in length

d) Time runs faster on the spaceship and it increases in length

17. [2003 HSC] An astronaut set out in a spaceship from Earth orbit to travel to a distant star in our galaxy. The spaceship travelled at a speed of 0.8 *c*. When the spaceship reached the star the on-board clock showed the astronaut that the journey took 10 years. An identical clock remained on earth. What time in years had elapsed on this clock when seen from the astronaut's spaceship? **[2 marks]**

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18. [2004 HSC] An object of rest mass 8.0 kg moves at a speed of 0.6*c* relative to an observer. What is the observed mass of the object? **[1 mark]**

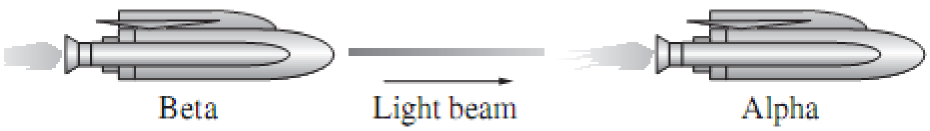
a) 6.4 kg

b) 10.0 kg

c) 12.5 kg

d) 13.4 kg

19. [2004 HSC] Two spaceships are both travelling at relativistic speeds. Spaceship Beta shines a light beam forward as shown. What is the speed of the light beam according to an observer on spaceship Alpha?



a) The speed of the light beam is equal to *c*.

b) The speed of the light beam is less than *c*.

c) The speed of the light beam is greater than *c*.

d) More information is required about the relative speed of the spaceships.

20. [2007 HSC] A spaceship sitting on its launch pad is measured to have a length L. This spaceship passes an outer planet at a speed of 0.95 *c*. Which observations of the length of the spaceship are correct? **[1 mark]**

	Observer on the spaceship	**Observer on the planet**
a)	No change	Shorter than L
b)	No change	Greater than L
c)	Shorter than L	No change
d)	Greater than L	No change

21. [2007 HSC]
    a) How has our understanding of time been influenced by the discovery of the constancy of the speed of light? **[2 marks]**

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    b) A piece of radioactive material of mass 2.5 kilogram undergoes radioactive decay. How much energy is released if 10 grams of this mass are converted to energy during the decay process? **[2 marks]**

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c) A mass is moving in an inertial frame of reference at a velocity v relative to a stationary observer. The observer measures an apparent mass increase of 0.37%. Calculate the value of v in ms^{-1}. **[3 marks]**

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22. [2008 HSC] A spaceship is travelling away from Earth at 1.8×10^8 m s^{-1}. The time interval between consecutive ticks of a clock on board the spaceship is 0.50 s. Each time the clock ticks, a radio pulse is transmitted back to Earth. **[1 mark]**

What is the time interval between consecutive radio pulses as measured on Earth?

a) 0.40s
b) 0.50s
c) 0.63s
d) 0.78s

23. Consider two synchronised atomic clocks, A and B. Clock A is placed on a spacecraft and clock B is left on Earth. The spacecraft then takes off and flies away at a speed of $0.99c$ relative to Earth.

Onboard the spacecraft is a powerful telescope that allows those onboard the spacecraft to observe clock B on Earth while the spacecraft is flying (this is theoretically possible, as light from Earth still reaches the spacecraft at exactly c).

a) For each second that passes on clock A, how many seconds pass on clock B? **[2 marks]**

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b) On Earth, there is also a powerful telescope that allows scientists to look at clock A while the spacecraft is flying. For each second that passes on clock B, how many seconds pass on clock A? **[2 marks]**

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Note: Did your answers to a) and b) seem to contradict each other? If they did, then your answers are likely to be correct. Time seemed to be slow on A relative to B, but also on B relative to A.

The reason for this apparent contradiction is: time cannot be compared between frames that are in relative motion. If the spacecraft turned around and flew back to Earth, clock B would actually rapidly ‘speed up’ relative to the spacecraft as the spacecraft decelerates, so when the spacecraft finally returns to Earth, only A was slow relative to B.

This situation is actually the same as the famous ‘Twin’s Paradox’. If you are interested, Wikipedia has a page called the ‘Twins Paradox’, which is a good introduction to the topic.

End of homework